Real-Time Constraints for Adaptive Digital Filters on MIMO Stochastic Systems

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Abstract. In this work, we expose the basic properties about the Real-time Adaptive Digital Filter on MIMO (Multi – Input, Multi - Output) case. This filter has a whole of conditions (in agreement to [1], [7] and [3]):
a) The number of vector operations required in the MIMO adaptive filter process by each iteration, is bounded with time constraints respect to dynamical process conditions [8] and,
b) The filter tasks vector are scheduled, and then the MIMO adaptive filter either implanted.
A MIMO digital filter is compound by two steps: the estimation parameter and identification states.
The filter MIMO implementation into PC require sets of dynamical arrays using the process message passing with priorities, without to lose the synchrony between the whole of tasks that compound its process algorithm.

Keywords: *Filter, estimation, identification, real-time, synchrony, time constraints.*

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1 Introduction

In [1] we say that a RTDF's are used in industrial processes, control systems and monitoring systems; for example: chemical plants, manufacturing processes, airbags and fuel injection systems, voice analysis, data acquisition, medical applications, telecommunications, missile trajectories, spatial rockets, and robotics operations. The Digital Filters cannot fail, in two senses: a) response quality and b) response time, in the other sense, the processes will be crash. For these motives, the Digital Filters properties in an implementation look up as Real-time Digital Filters (RTDF): it will use into embedded systems using electronic devices or digital computers with Real-time Operating Systems (RTOS). However, all of them, require evaluating their answers with respect to desired output, these difference considered into the basic law (that bound the gains used inside of theirs) establish the high converge regions, this kind of filters is known as Real-time Adaptive Digital Filters (RTADF) in MIMO case (RTMADF).

A huge problem implementation is developed when a RTMADF is used: in practical sense, the real world is parallel (to see references in [1]) and it has many variables in interacting in it, then a Real-time Multivariable Adaptive Digital Filter practically is required, if and only if the time evolution corresponds basically to the time system evolution. In this case, generality the users observes other problem, when the computing resources are limited in operations or in time output interaction considering a lot of variables that requires computer resources with time restrictions. The concurrent task synchronized by message passing and precedence constraints is considered if we have a processor.

2 Real-Time Multivariable Adaptive Digital Filter (RTMADF)

The class of digital filters applied into PC and that interacts with dynamic processes and emits high quality output responses with time constraints and critical synchrony, will be described as RTADF (Real-time Adaptive Digital Filters). In [2], [3], [4], [5], and [6] exposing that Real-time Adaptive Digital Filters (Kalman Filter for example) can be implanted as a RTS considering the high velocity computers and facility to express the filter in recursive form. Considering in it the times measure of all tasks around the implementation of it into PC (considering the measurable: The A/D and D/A converters, processor operations, filter algorithm and precedence constraints). In [7] was exposed the definition of Real-time digital filter, considering in it the properties previously cited in this paragraph, all in SISO (Single-Input Single-Output) representation.

The basic properties of RTADF in SISO representation is the base required for to describe a Real-time adaptive digital filter in MIMO (Multi-Input Multi Output) sense. The follows definition exposes the properties of it.

Definition 1. A RTMADF is a Multivariable Adaptive Digital Filter with time constraints in the Nyquist Multivariable sense, in according to dynamical inputs and convolution feedback law filter, synchronously interacting both, accomplishing the conditions:

a. To receive and to give responses (where $\{u(k)_i \in \mathrm{U}(k)\}$, $\{e(k)_{i-j} \in E(k)\}$ and $\{y(k)_j \in \mathrm{Y}(k)\}$, with $i, j, k \in Z^+$, are the input, the feedback and, output responses, respectively.) of a real dynamical process in synchronized time, bounded them in the Nyquist Multivariable sense (to see in simple sense: [27]).
b. To express the RTMADF in recursive form, describing on line, the physical process evolution considered,
c. To give good enough responses in time with respect to dynamical process: All vector operations required into recursive and adaptation form in each iteration system evolution, are bounded at time dynamically considering the process constraints,
d. The convergence rate value is bounded into a finite interval around the real value, converging to the upper limit of the noise interacting with the system considered.
e. A RTMADF has a concurrent tasks vector, considering that the scheduling of all instances and operations around of recursive and adaptive filter requirements are bounded by unique processor operations with respect to input and output responses desired.

A RTMADF symbolically defined as $(\{u(k)_i \in \mathrm{U}(k)\}, \{e(k)_{i-j} \in E(k)\}, \{y(k)_j \in \mathrm{Y}(k)\})$ with $i, j, k \in Z^+$, bounded each set in the Nyquist multivariable sense, considering that the sampling time is obtained as a minimal distance into the control area, symbolically is expressed as:

$$t_{sample_{MIMO}} = \min\left(\left(t_{sample}\left(u(k)_i\right) \circ t_{convolution}\left\{e(k)_{i-j}\right\}\right), t_{sample}\left(y(k)_j\right)\right), \tag{1}$$

into interval k. For example, considering the second minimum sample period into the set of all sample periods of MIMO system, then the signal with largest change velocity is lost by aliasing, it cause that filter do not describe correctly the dynamical evolution of process. It's a contradiction and, consequently is required to select the shortest sample time into the control area: Basically, this area is conformed by a set of points, with relative distance between them and absolute distance with respect to the reference point, where the minimal absolute distance is contained in all control area points.

The system should have two conditions: a) $t_{sample_{MIMO}}$ selected, is lowest Euclidian distance into the temporized control area (1), and hasn't lost information by aliasing, b) it has a $t'_{sample_{MIMO}} \subset Y(k)$ lower than $t_{sample_{MIMO}}$ selected previously in the same interval time, indexed by k. In this case, the filter doesn't have defined its control area and has negative aliasing, and the signal has perturbations, losing information.

In this case each sampling time in this kind of filters has a basic measures:
$t_{sample}(u(k)_i) = \mu[t_{sample}(u(k)_i), \overline{t_{sample}(u(k)_i)})$,
$t_{convolution}(e(k)_i) = \mu[t_{convolution}(e(k)_i), \overline{t_{convolution}(e(k)_i)})$ and
$t_{sample}(y(k)_j) = \mu[t_{sample}(y(k)_j), \overline{t_{sample}(y(k)_j)})$ (to see: [9]), where
$t_{sample}(u(k)_i) \le 0.5(F_{\max}(u(k)_i))^{-1}, t_{convolution}(e(k)_{i-j}) \le 0.5(F_{\max}(E(k)_{i-j}))^{-1}$ and
$t_{sample}(y(k)_j) \le 0.5(F_{\max}(Y(k)_{i-j}))^{-1}$ in agreement to [8] and [27].

In the shortest sampling time $t_{sample_{MIMO}}$ includes the minimal sample times input set $U(k)$, sample times feedback law set $E(k)$ and sample times output set $Y(k)$. But the maximal times distance into the control area previously defined, it doesn't highest that the times evolution in the Multivariable Nyquist concept, losing synchrony, because the response times are out of the intervals indexed with k, defined by $t_{sample_{MIMO}}$ evolution delaying and displacing its interactions, obtaining a queue old problem.

Definition 2. The time constraint vectors of RTMADF is compound by the follows vectors set (to see: [10]): Arrival time ($\mathbf{l}(k)$)., Computation time ($\mathbf{C}(k)$)., Minimum relative deadline ($\mathbf{D}_{\min}(k)$): The minimum absolute deadline is $\mathbf{ld}_{\min}(k) = \mathbf{l}(k) + \mathbf{D}_{\min}(k)$., Maximum relative deadline ($\mathbf{D}_{\max}(k)$): The maximum absolute deadline is $\mathbf{LD}_{\max}(k) = \mathbf{l}(k) + \mathbf{D}_{\max}(k)$., Start time ($\mathbf{s}(k)$)., Finishing time ($\mathbf{f}(k)$)., Lateness ($\mathbf{L}(k)$)., Premature time ($\mathbf{P}(k)$)., Interaction Period (T) is $t_{sample_{MIMO}}$., Convergence time (t_c), such as $d_{c\_\min} \le t_c < d$ where $d_{c\_\min}$ and, d have a minimal convergence deadline and convergence deadline, respectively.

Each time constraint vector have a set of elements such that describing to the set of tasks to build a RTMADF.

Commonly, arrival time ($\mathbf{l}(k)$), start time ($\mathbf{s}(k)$) and finishing time ($\mathbf{f}(k)$) are absolutes and the others times are relatives.

The computation time ($\mathbf{C}(k)$) defined as $\mathbf{C}(k) := \mu[\mathbf{s}(k), \mathbf{f}(k))$ and considering [9], expressed as $\mathbf{C}(k) = \mathbf{f}(k) - \mathbf{s}(k)$, where both absolute times are increasing in time evolution, i.e., $f(k) \uparrow f(m)$ *and* $s(k) \uparrow s(m)$, respectively; where the index interval described by m indicate the convergence of the RTMADF with respect to Definition 1. Nevertheless, other side, computation time vector $\mathbf{C}(k)$ either defined by the measure of all segments of time disjoints describe these in symbolic form by the set $\{C(k)_r\}$ where each $C(k)_r$ is defined in the same metric as $\mu[s(k)_r, f(k)_r)$ $\forall r \in Z^+$ accomplishing that $\sum_{i=\overline{1,n}} C(k)_i < t_{sample_{MIMO}}$. Observing that $C(k)_r = \mu[s(k)_r, f(k)_r)$ $\forall r \in Z^+$, and considering that $[s(k)_r, f(k)_r)$

$\bigcap [s(k)_m, f(k)_m) = \phi \quad \forall r \neq m$ then $\mu(\bigcup_{r=\overline{1,n}} [s(k)_r, f(k)_r) = \sum_{r=\overline{1,n}} [s(k)_r, f(k)_r)$, in other words $\sum_{r=\overline{1,n}} \mu([s(k)_r, f(k)_r)) = \sum_{r=\overline{1,n}} C(k)_r$, but $C(k) = \sum_{r=\overline{1,n}} C(k)_r$ because $\mu([s(k), f(k))) = \sum_{r=\overline{1,n}} \mu([s(k)_r, f(k)_r))$ and $\mu([s(k), f(k))) < t_{sample\ MIMO}$, i.e., $\sum_{r=\overline{1,n}} C(k)_r < t_{sample\ MIMO}$.
Finally in this section, the RTMADF has a Convergence time, expressed symbolically by (t_c) and is a maximum scalar value defined as $t_c := \max\{f(k = m_{\max})_i\}$ with $\{f(k = m_{\max})_i\} \subseteq f(m)$, such that in other case don't converge yet, considering the previous concepts, exposed before definitions.

3 Multivarible Adaptive Digital Filter considering the Instrumental Variable as Estimator and Kalman Filter as Identificator

A classical problem in that used a RTMADF is a time-varying parameter estimation, consider a real dynamic process and the necessity of estimate in Real-time its parameters (the best gains required into the Kalman filter identificator). The most immediate and simple idea is to include a discounting procedure in an estimation algorithm i.e., a procedure for discarding (forgetting) old information. The most common way to do is to introduce an exponential forgetting factor (FF) into the corresponding estimation procedure in which use an instrumental variable (to see: [11]). A large group of papers deals with constant scalar FF, for example: In [12] shown that, when a FF applied to the past data, the convergence exponentially bounded, and prediction coefficients fluctuate around the least squares estimates in the steady state, in probability sense. A small forgetting factor increases the convergence rate, but results have a larger fluctuation around of real values and convergence time is increased, to see: [13]; these authors present a theoretical analysis of the performance of the standard FF in a recursive least squares algorithm, they used it combination in the tracking of time-varying linear regression models. As application of it, [14] considers it with adaptive techniques to control the space station in orbit with moving payload.
In [15] is presented a modified sequential least squares parameter estimation method and compares its performance to that of standard least square method techniques using a simulated nonlinear F-16 with multiaxial thrust-vectoring aircraft. It is shown that the suggested method offers significant improvement in performance over the conventional least squares method parameter estimation by providing a recursive estimation algorithm that penalizes noisy estimates and is less subject to ill-conditioning as its FF is reduced.
The main characteristic of these algorithms in agreement to the set of researcher cited above ([16], [17], [18], [19], [20], [21], [22], [23], [24], [25], and [26], in the others) is to permit to estimate through of scalar exponential FF, the nonstationary dynamics of system.
In [27] suggested a new approach based on the use of the recursive version of the Instrumental Variable Method (IVM) with a constant Matrix FF (MFF) providing the best estimation quality than a scalar one and espoused the properties of matrix

variable FF (VFF) according to the current prediction error. The error convergence analysis was development and evaluated with respect to least square method, both with MFF, where the IVM is approximately two times with respect to LSM, evaluated these in probability sense.
The model considered (to see: [28], [29]) is expressed in symbolic form in (2)

$$d\mathbf{x}_t = [\mathbf{A}(\omega_t')\mathbf{x}_t + \mathbf{B}u_t]dt + \mathbf{D}\xi_t \tag{2}$$

Where the vector state is expressed as:

$$\mathbf{x}_t := [i_{a_t}\ i_{b_t}\ \varphi_{a_t}\ \varphi_{b_t}] \in R^4 \tag{3}$$

The state space vector containing bounded signals

$$\mathbf{U}_t := [u_{a_t}\ u_{b_t}]^T \in R^2 \tag{4}$$

ξ_τ is defined as Standard Brownian motion process. The matrices $\mathbf{A}(\omega_t')$ and $\mathbf{B}$ and the vector $\mathbf{D}$, are defined as follows:

$$\mathbf{A}(\omega_t') = \begin{bmatrix} -\gamma & 0 & \frac{K}{T_r} & p\omega_t' K \\ 0 & -\gamma & -p\omega_t' K & \frac{K}{T_r} \\ \frac{M}{T_r} & 0 & -\frac{1}{T_r} & -p\omega_t' \\ 0 & \frac{M}{T_r} & p\omega_t' & -\frac{1}{T_r} \end{bmatrix} \tag{5}$$

$$\mathbf{B} = \begin{pmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 0.000573 \\ 0.000457 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

Here ω' is varying according to the following dynamics:

$$d\omega_t' = K_I \mathrm{H}_\tau - K_{II} \tag{7}$$

$$K_I = \frac{pM}{JL_r},\ K_{II} = \frac{T_l}{J},\ \mathrm{H} = \varphi_{a_t} i_{b_t} - \varphi_{b_t} i_{a_t} \tag{8}$$

$$\omega_0' = 0.01 s^{-1} \tag{9}$$

All parameters participating in the description of the dynamic model (2) are used in agreement to [10]. With the follows gains:

$$K = \frac{M}{\sigma L_s L_r},\quad \gamma = \frac{\mathrm{R}_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r^2}, \tag{10}$$

$$\sigma = 1 - \left(\frac{\mathrm{M}^2}{\sigma L_s L_r^2} \right) \tag{11}$$

Selecting the time sequence t_τ as:

$$t_{\tau+1} = t_\tau + \Delta_\tau,\ \Delta_\tau = 0.015\mathrm{s} \tag{12}$$

The expression (2) has in discrete time the following space state:

$$\mathbf{x}_{\tau+1} = \mathbf{A}_\tau \mathbf{x}_\tau + \Theta_\tau \zeta_\tau' \tag{14}$$

Where $\mathbf{A}_\tau = [\mathbf{I} + \mathbf{A}(\omega_\tau)\Delta_\tau]$, $\Theta_\tau = [\mathbf{B}\Delta_\tau \;\; \mathbf{D}]$, $\zeta_\tau' = [\mathbf{U}_\tau \;\; \xi_\tau']$ With:

$$\omega_{\tau+1}' = \omega_\tau' + [K_I \mathrm{H}_\tau - K_{II}]\Delta_\tau \tag{15}$$

The random variable ξ_τ' in differences has the form:

$$\xi_\tau' = \xi_{\tau+1} - \xi_\tau \tag{16}$$

The output signal described as $\mathbf{Y}_\tau$, has in discrete time the symbolic expression:

$$Y_\tau = A_\tau Y_{\tau-1} + \theta_\tau \zeta_\tau \tag{17}$$

The problem is to generate an optimal estimation with respect to $\mathbf{A}_\tau$ at real-time in agreement to both definitions, requiring a good enough answer with respect to real modeled system. The estimator was described in [27], where the recursive form of it is expressed as:

$$\hat{\mathbf{A}}_\tau = \hat{\mathbf{A}}_{\tau-1} + (\mathbf{I} - \hat{\mathbf{A}}_{\tau-1}\mathbf{z}_\tau)\vartheta_\tau^T \mathbf{\Gamma}_\tau,$$

$$\mathbf{\Gamma}_\tau = \mathbf{R}^{-1}\mathbf{\Gamma}_{\tau-1}\frac{\mathbf{R}^{-1}\mathbf{\Gamma}_{\tau-1}\mathbf{z}_\tau\vartheta_\tau^T\mathbf{R}^{-1}\mathbf{\Gamma}_{\tau-1}}{1+\vartheta_\tau^T\mathbf{R}^{-1}\mathbf{\Gamma}_{\tau-1}\mathbf{z}_\tau} \tag{18}$$

The time evaluation of the corresponding performance index J_n given by

$$J_n = \frac{1}{n}\sum_{\tau=1}^{n} tr\{(\hat{\mathbf{A}}_\tau - \mathbf{A}_\tau)(\hat{\mathbf{A}}_\tau - \mathbf{A}_\tau)^T\} \tag{19}$$

Now, the Kalman filter identificator has the following structure:

$$\hat{y}_{n/Y_{n-1}} = \hat{A}_n \hat{x}_{n/Y_{n-1}} + \theta_\tau \zeta \tag{20}$$

With the innovation process described into adaptive sequence by $\alpha_n = y_n - \hat{y}_{n/Y_{n-1}}$, the matrix $G_n = \hat{A}_\tau K_{n,n-1}\Sigma_n^{-1}$, and the adaptive error defined by the second probability moment $K_{n,n-1} = E[\varepsilon_{n,n-1}\varepsilon_{n,n-1}^H]$, considering that $\varepsilon_{n,n-1} = x_n - \hat{x}_{n/Y_{n-1}}$, $Q_2 = K_{n,n-1}$, and $\Sigma_n = K_{n,n-1} + Q_2$.

The internal identificator described in adaptive sense, has the form

$$\hat{x}_{n+1/Y_n} = \hat{A}_\tau \hat{x}_{n/Y_{n-1}} + G_n \alpha_n \tag{21}$$

The real time multivariable adaptive filtering has the optimal forgetting factor matrix based in [27]:

$$R^* = \begin{bmatrix} 0.99989 & 0.0009 & 0.0009 & 0.0005 \\ 0.0009 & 0.99898 & 0.0003 & 0.0007 \\ 0.0009 & 0.0003 & 0.9999 & 0.0002 \\ 0.0005 & 0.0007 & 0.0002 & 0.89898 \end{bmatrix} \tag{22}$$

4 RTMADF Implementation (Dc - Derivate Motor Description)

The time-varying matrix with respect to ω_τ' using the algorithm expressed in (18) with (22), and the adaptive Kalman filter using the algorithm expressed from (20) to

(22) implemented in QNX RTP Ver. 6.0 using the following hardware and software elements: **Hardware:** A D.C. Motor, with follows properties: $20\ V$, $1\ A$, $18000\ rpm$, two poles, with permanent field., Power unit A/D, D/A, $5\ V$ input, $20\ V$ output, $0.05\ A$ input, $5\ A$ output., PC Pentium III, $400\ MHz$, $64\ MB$., A/D D/A card Advantech PCL 818L. **Software:** QNX RTP 6.0 Real-time Operating system. MicroPhoton Development Kit., Watcom C 10.0.

The time evaluation of the time varying parameters using (18) depicted in Fig 1, illustrate the time of convergence of it to the real evolution and the functional using (19) has the high level of convergence and is depicted in Fig 2:

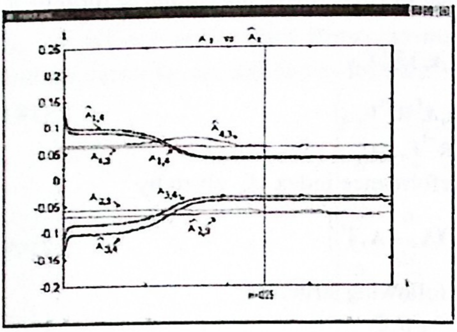

Fig 1. RTMADF (Parameter estimator)

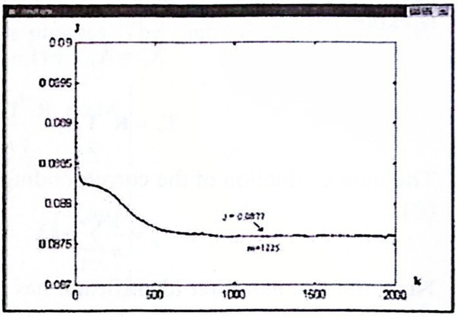

Fig 2. Time evaluation of J_n.

Finally, the simulation results by Kalman filter as RTMADF (was used the expressions (20) and (21)) has the high level of performance and has too the equal time of convergence with respect to the real system modeled by (14),

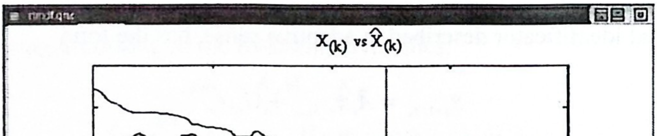

Fig 3. Time evaluation of $x_n - \hat{x}_{n/Y_{n-1}}$.

These simulations showed that the RTMADF accomplished in the same interval the best results in a convergence sense considering in it the expressions (18) to (22).

5 CONCLUSIONS

In this work, we have exposed the basic concepts about the real-time multivariable adaptive digital filter (RTMADF).
The basic concepts considered were the properties about the real-time systems, and adaptive digital filters, both in multivariable sense; such that in agreement to [1], [7] and [3], required us the vector properties with respect to:
a) The number of vector operations required in the multivariable adaptive filter process by each iteration, bounded with time constraints respect to dynamical process conditions [8] and,
b) The filter tasks vector were scheduled, such that the adaptive filter could be implanted.
With respect to Multivariable Adaptive Digital Filter, we said that it was compound by two steps, described these as: the estimation and identification sections, developed in section 3, considering the Instrumental Variable and Kalman Filter by no stationary output system, respectively.
The RTMADF implementation was solved and either illustrated using a DC derivate multivariable novel model. In this case, we were requiring the dynamical arrays sets with message passing priorities, without to lose the synchrony between the whole of tasks vector.

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